Assignment

1. Write each expression as a single logarithm:

a) $\log x - 3\log y - 2\log z$

$= \log x - \log y^3 - \log z^3$

$= \log x - (\log y^3 + \log z^3)$

$= \log x - (\log(y^3 z^3))$

$= \log\left(\frac{x}{y^3 z^3}\right)$

b) $\frac{1}{3}\log_a p + 3\log_a q - 4\log_a p$

$= \log_a p^{1/3} + \log_a q^3 - \log_a p^4$

$= \log_a\left(\frac{p^{1/3} q^3}{p^4}\right) = \log_a\left(\frac{q^3}{p^{11/3}}\right)$

$p^{1/3} - p^4 = p$

2. Simplify the following without using a calculator.

a) $\log 2 + 2\log 3 - \log 18$

$= \log 2 + \log 3^2 - \log 18$

$= \log 2 + \log 9 - \log 18$

$= \log(2 \cdot 9) - \log 18$

$= \log 18 - \log 18$

$= \underline{\underline{0}}$

b) $2\log_4 2 - 2\log_4 4 - \log_4 \frac{1}{4}$

$= \log_4 2^2 - \log_4 4^2 - \log_4 \frac{1}{4}$

$= \log_4 4 - \log_4 16 - \log_4 \frac{1}{4}$

$= \log_4 4 - (\log_4 16 + \log_4 \frac{1}{4})$

$= \log_4 4 - (\log_4(16 \cdot \frac{1}{4}))$

$= \log_4 4 - \log_4 4$

$= 0 //$

3. Use the laws of logarithms to simplify and evaluate the following expressions.

a) $\log_2\sqrt{6} - \frac{1}{2}\log_2 3$

$= \log_2\sqrt{6} - \log_2 3^{1/2}$

$= \log_2\sqrt{6} - \log_2\sqrt{3}$

$= \log_2\left(\frac{\sqrt{6}}{\sqrt{3}}\right)$

$= \log_2\sqrt{2}$

$= \log_2 2^{1/2}$

$= \frac{1}{2}\log_2 2$

$= \frac{1}{2} \cdot 1 = 1/2$

b) $\frac{1}{2}\log_{10}10 + 3\log_{10}\sqrt{10}$

$= \frac{1}{2}\log_{10}10 + 3\log_{10}10^{\frac{1}{2}}$

$= \frac{1}{2}\log_{10}10 + 3 \cdot \frac{1}{2}(\log_{10}10)$

$= (\frac{1}{2} \cdot 1) + 3(\frac{1}{2})(1)$

$= \underline{\underline{2}}$

4. Simplify the following:

a) $\log x^4 - 3\log x + \log \frac{1}{x}$

$= 4\log x - 3\log x + \log 1 - \log x$

$= \log 1$

$= 0$

b) $\log x^{\frac{1}{2}} + \log y^{\frac{1}{2}} - \frac{1}{2}\log xy$

$\frac{1}{2}\log x + \frac{1}{2}\log y - \frac{1}{2}(\log x + \log y)$

$= \frac{1}{2}\log x + \frac{1}{2}\log y - \frac{1}{2}\log x - \frac{1}{2}\log y$

$= 0$

c) $\log_a a^{2x+1} - \log_a a^{x-7}$

$= (2x+1)\log_a a - (x-7)\log_a a$

$= (2x+1)(1) - (x-7)(1)$

$= 2x+1-x+7$

$= x+8$

d) $\log_2 a^{x+5} + 2\log_2 a^{x-3}$

$= \log_2 a^{x+5} + \log_2 a^{2(x-3)}$

$= \log_2 a^{x+5} + \log_2 a^{2x-6}$

$= \log_2 (a^{x+5} \cdot a^{2x-6})$

$x+5+2x-6$
$= 3x-1$

$= \log_2 a^{3x-1}$ or $(3x-1)\log_2 a$.

5. Show that $\log_a y^{2x-3} + \log_a y^{5x-2} - \log_a y^{x-5} - 2\log_a y^{3x+1}$ can be written as $\log_a\left(\frac{1}{y^2}\right)$

$= (2x-3)\log_a y + (5x-2)\log_a y - (x-5)\log_a y - 2(3x+1)\log_a y$

$= \log_a y\left[(2x-3) + (5x-2) - (x-5) - 2(3x+1)\right]$

$= \log_a y\,(2x-3+5-2-x+5-6x-2)$

$= \log_a y\,(-2)$

$= -2\log_a y$

$= \log_a \underbrace{y^{-2}}_{\frac{1}{y^2}} = \log_a\left(\frac{1}{y^2}\right)$

6. Determine the value of the following.

a) $(5^{\log_5 2})(5^{\log_5 3})$ . . b) $\dfrac{\left(\sqrt{2}^{\ \log_6 27}\right)\left(\sqrt{2}^{\ \log_6 16}\right)}{\sqrt{2}^{\ \log_6 12}}$

$= 5^{\log_5 2 + \log_5 3}$

$= 5^{\log_5 (2 \cdot 3)}$

$= 5^{\log_5 6}$

$= \underline{6}$

$= \sqrt{2}^{\ \log_6 27 + \log_6 16 - \log_6 12}$

$= \sqrt{2}^{\ \log_6 \left(\frac{27 \times 16}{12}\right)} = \sqrt{2}^{\ \log_6 36}$ $\rightarrow$ $\log_6 36 = \underline{\underline{2}}$

$= (\sqrt{2})^2 = \underline{\underline{2}}$

Multiple Choice

7. The expression $3 \log_x 4 + \log_x 8 - \frac{1}{4} \log_x 16$, where $x > 0$, is equal to

A. $\log_x 384$

B. $\frac{3}{4} \log_x 512$

C. $\log_x 256$

D. $\frac{1}{4} \log_x \left(\frac{1}{2}\right)$

$= \log_x 4^3 + \log_x 8 - \log_x 16^{\frac{1}{4}}$ $\rightarrow$ $\sqrt[4]{16} = 2.$

$= \log_x 64 + \log_x 8 - \log_x 2$

$= \log_x \left(\frac{64 \cdot 8}{2}\right)$

$= \log_x 256.$

8. $\log_p(p^6 q^2) - \log_p(p^2 q^2)$ is equivalent to

A. 3

B. 4

C. $4p$

D. p^4

$\log_p \left(\frac{p^6 q^2}{p^2 q^2}\right)$

$\log_p (p^4)$

$= 4 \log_p p = 4(1) = \underline{4}.$

9. If $\log_3 A = t$, then $\log_3 27A^3 =$

A. $3 + 3t$

B. $3 + t^3$

C. $9t^2$

D. $3t^3$

$= \log_3 27 + \log_3 A^3$

$= 3 + 3 \log_3 A$

$= 3 + 3t$

Numerical Response

10. If $\log_3 x^2 = 2$ and $2\log_k \sqrt{x} = \frac{1}{3}$, then the value of k is _____ .

(Record your answer in the numerical response box from left to right.)

2	7		

$\log_3 x^2 = 2$

$x^2 = 3^2$

$x^2 = 9$

$x = \pm 3$

$2\log_k \sqrt{3} = \frac{1}{3}$

$\log_k (\sqrt{3})^2 = \frac{1}{3}$

$\log_k 3 = \frac{1}{3}$

$3 = k^{1/3}$

$3^{\frac{3}{1}} = k^{\frac{1}{3}\cdot\frac{3}{1}}$

$3^3 = k = 27.$

Numerical Response

11. If $\log_3 x^2 = 4$, $\log_2 y^3 = 6$, and $\log_b x + \log_b y = \frac{1}{2}$, where $x, y > 0$, then the value of b is _____ .

① Solve for x+y

(Record your answer in the numerical response box from left to right.)

1	2	9	6

$\log_3 x^2 = 4$

$x^2 = 3^4$

$x^2 = 81$

$x = \pm 9$

$x = 9$

$(x > 0)$

$\log_2 y^3 = 6$

$y^3 = 2^6$

$y^3 = 64$

$y = 4$

$\log_b x + \log_b y = \frac{1}{2}$

$\log_b 9 + \log_b 4 = \frac{1}{2}$

$\log_b (9\cdot 4) = \frac{1}{2}$

$\log_b 36 = \frac{1}{2}$

$36 = b^{1/2}$

$36^{\frac{2}{1}} = b^{\frac{1}{2}\cdot\frac{2}{1}}$

$1296 = b$

Answer Key

1. a) $\log\left(\frac{x}{y^3 z^2}\right)$ **b)** $\log_a\left(\frac{q^3}{p^{\frac{11}{3}}}\right)$ **2. a)** 0 **b)** 0 **3. a)** $\frac{1}{2}$ **b)** 2

4. a) 0 **b)** 0 **c)** $x + 8$ **d)** $\log_2 a^{3x-1}$ or $(3x-1)\log_2 a$

6. a) 6 **b)** 2 **7.** C **8.** B **9.** A

10.

2	7		

11.

1	2	9	6