

b.) @ 1100°C only  $\gamma$  exists W <sub>$\gamma$</sub>  = 1 ~~+2~~ +4

@ 722°C (just below the eutectoid)

+3

$$W_{\alpha} = \frac{6.7 - 0.5}{6.7 - 0.045} + 1 \quad W_{Fe_3C} = \frac{0.5 - 0.045}{6.7 - 0.045} + 1$$

$$W_{\alpha} = 0.9316 + 5 \quad W_{Fe_3C} = 0.0684 + 5$$

@ 500°C

+3

$$W_{\alpha} = \frac{6.7 - 0.5}{6.7 - 0} + 1 \quad W_{Fe_3C} = \frac{0.5 - 0}{6.7 - 0} + 1$$

$$W_{\alpha} = 0.9254 + 5 \quad W_{Fe_3C} = 0.0746 + 5$$

c.) 2 possible answers:

\* best answer: 723°C when  $\gamma$  becomes 0.77% C from our starting Fe - 0.5 wt% C melt

\* acceptable answer: the austenite phase can have the highest carbon content (solubility limit temperature for  $\gamma$ ) at 1147°C when  $\gamma$  has ~ 2% Carbon.

+5

$$\frac{0.77 - 0.5}{0.77 - 0.045} + 1 = 0.3724 + 5$$

+5



$$\textcircled{2} \quad \frac{C_x - C_s}{C_s - C_0} = 1 - \text{erf}\left(\frac{x_1}{2\sqrt{D_1 t_1}}\right) = 1 - \text{erf}\left(\frac{x_2}{2\sqrt{D_2 t_2}}\right)$$

recognize that different treatments must yield same result given the amount of time at the correct temperature.

thus:  $x_1 = x_2 = x$

and  $\text{erf}\left(\frac{x}{2\sqrt{D_1 t_1}}\right) = \text{erf}\left(\frac{x}{2\sqrt{D_2 t_2}}\right)$

+6  
equate the two

so  $D_1 t_1 = D_2 t_2 \Rightarrow D_1 = \frac{t_2}{t_1} D_2$

$D_1 = D_0 \exp\left(\frac{-Q}{RT_1}\right) = \frac{t_2}{t_1} D_0 \exp\left(\frac{-Q}{RT_2}\right)$

+6 diffusion equation

so say:  $D_0 = 2.3 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$

$t_1 = 2 \text{ hours}$

$R = 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}$

+3  
proper units

$T_1 = \boxed{\text{the answer?}}$

$k_B \cdot N_A = R$

$Q = 148,000 \frac{\text{J}}{\text{mol}}$

$t_2 = 15 \text{ hours}$

convert units

$T_2 = 900 \text{ C} = 1173 \text{ K}$

+4

$$D_0 \exp\left[\frac{-Q}{RT_1}\right] = \frac{t_2}{t_1} D_0 \exp\left[\frac{-Q}{RT_2}\right]$$

~~plug in~~

$$\ln\left[\exp\left[\frac{-Q}{RT_1}\right]\right] = \ln\left[\frac{t_2}{t_1} \exp\left[\frac{-Q}{RT_2}\right]\right]$$

$$\frac{-Q}{RT_1} = \ln\left[\frac{t_2}{t_1} \exp\left[\frac{-Q}{RT_2}\right]\right]$$

+4  
solve

rearrange to

get

$$T_1 = \frac{-Q}{R \ln\left[\frac{t_2}{t_1} \exp\left[\frac{-Q}{RT_2}\right]\right]}$$

plug in  $-148,000 \frac{\text{J}}{\text{mol}}$

$$= \frac{8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \ln\left[\frac{15}{2} \exp\left[\frac{-148000 \frac{\text{J}}{\text{mol}}}{8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \cdot 1173 \text{ K}}\right]\right]}{-148000 \frac{\text{J}}{\text{mol}}}$$

$T_1 = 1352.6 \text{ K} = 1079.6 \text{ C}$

+2 answer



25 total points

Problem 3:

Calculate the energy for vacancy formation in silver, given that the equilibrium number of vacancies at 800°C (1073K) is  $3.6 \times 10^{23} \text{ m}^{-3}$ . The atomic weight and density at 800°C are 107.9 g/mol and 9.5 g/cm<sup>3</sup>. (Avogadro's number =  $6.022 \times 10^{23}$  atoms per mol.)

(+6)  $c=1$   
(+6)  $n_v = n_0 \exp\left(-\frac{E_{\text{Form}}}{kT}\right) \Rightarrow kT \ln\left(\frac{n_v}{n_0}\right) = -E_{\text{Form}}$  (+5)

(+6)  $n_0 = \frac{N_A \left(\frac{9.5 \text{ g}}{\text{cm}^3}\right)}{\left(\frac{107.9 \text{ g}}{\text{mol}}\right)} \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 5.302 \times 10^{28} \text{ m}^{-3}$

$\Rightarrow E_{\text{Form}} = -\left(1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}\right)(1073 \text{ K}) \ln\left(\frac{3.6 \times 10^{23} \text{ m}^{-3}}{5.302 \times 10^{28} \text{ m}^{-3}}\right)$

(+2)  $= 1.7629 \times 10^{-19} \text{ J} = 1.1 \text{ eV}$

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23  
17  
6

Problem 4:

Your sample of silver has a melting temperature of 1234.15K. After melting your sample you cool it to a temperature of 973.15K. Through the cooling process you determine that the latent heat of fusion  $\Delta E = \Delta H = .11 \text{ J/m}^3$ . You determine that your sample has a density of nucleation sites,  $s = 10^{26} \text{ m}^{-3}$  and the number of nuclei to be  $N = 1.06 \times 10^{24} \text{ m}^{-3}$ .

Knowing that the number of nuclei obeys this relation:

$$N = se\left(-\frac{\Delta G^*}{kT}\right)$$

7pts (a) Find  $\Delta G^*$ .

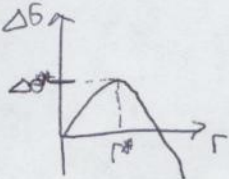
11pts (b) What is the value of the surface energy? (Assume a spherical particle)

7pts (c) What is the value of the critical radius  $r^*$ .

(a)  $\Delta G^* = -kT \ln\left(\frac{N}{s}\right) = -(1.38 \times 10^{-23} \frac{\text{J}}{\text{K}})(973.15 \text{ K}) \ln\left(\frac{1.06 \times 10^{24}}{10^{26}}\right)$  from

$\Rightarrow \Delta G^* = 6.109 \times 10^{-20} \text{ J} = .381 \text{ eV}$

(b) if  $r^* = \frac{-2\gamma}{\Delta G_V}$  is known then we can plug it into  $\Delta G = \frac{4}{3}\pi r^3 \Delta G_V + 4\pi r^2 \gamma$  to find  $\Delta G^*$



$r^*$  can be found by finding the maximum of  $\Delta G$  i.e.  $\frac{d\Delta G}{dr} = 0$  at  $r^*$

furthermore, it is known that

$$\Delta G_V = \frac{\Delta H \Delta T}{T_m}$$

at the melting temp.  $\Delta G_V = 0 = \Delta H - T_m \Delta S$

$\Rightarrow \Delta S = \frac{\Delta H}{T_m}$

$\Delta G_V = \Delta H - T \Delta S$

$= \Delta H \left(1 - \frac{T}{T_m}\right) = \frac{\Delta H \Delta T}{T_m}$

$\Rightarrow \Delta G^* = \frac{4}{3}\pi \left(\frac{-2\gamma}{\Delta G_V}\right)^3 \Delta G_V + 4\pi \left(\frac{-2\gamma}{\Delta G_V}\right)^2 \gamma$

Reduces to

$\Delta G^* = \frac{16\pi \gamma^3}{3\Delta G_V^2} \Rightarrow \gamma = \left(\frac{3\Delta G^* \Delta G_V^2}{16\pi}\right)^{1/3}$

$\Delta G_V = \frac{.11 \text{ J}}{\text{m}^3} (261 \text{ K}) \frac{T_m}{T_m}$

$1234.15 \text{ K}$

$= .023 \text{ J/m}^3$

$\Rightarrow \gamma = 1.25 \times 10^{-8} \frac{\text{J}}{\text{m}^2}$



(c)

$$\Gamma^* = \frac{\frac{-2.8}{\Delta G_V}}{\frac{-2.8 T_m}{\Delta H_{OT}}} = \frac{2(1.25 \times 10^{-8} \frac{J}{m^2})(1234.15 K)}{(.11 \frac{J}{m^2})(261 K)} = \frac{1.078 \mu m}{1.078 \times 10^{-6} m}$$